

Generalised scalar-tensor theory and the cosmic acceleration

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Abstract

In this paper it has been shown that a simple functional form of $\omega(\phi)$ in a generalised scalar tensor theory can drive the present cosmic acceleration without any quintessence field or the cosmological constant Λ . Furthermore, it ensures a smooth transition from a decelerated to an accelerated phase of expansion in the matter dominated regime.

1 Introduction

Although high precision observational data and their interpretations point towards an accelerated expansion of the universe with more and more certainty[1], the search for the ‘dark energy’ sector, which drives this acceleration, has not seen a preferred direction as yet. The good old cosmological constant Λ , the most proclaimed candidate as the source for this repulsive gravitational effect, does fit the observational data reasonably well, but it has its own problems[2]. Naturally a large number of other alternatives have already appeared in the literature with their own virtues and shortcomings[3].

Albeit their problems with local astronomical experiments, non-minimally coupled scalar field theories, particularly in the framework of Brans-Dicke (BD) theory, have proved to be useful in negotiating this counter-intuitive acceleration. It has been shown that BD theory along with a quintessence scalar field can indeed generate an accelerated expansion of the universe[4]. A variation of BD theory, for example an addition of a potential V which is a function of BD scalar field itself, can drive this desired accelerated expansion[5]. Most of these models suffer from two important drawbacks. One is that in these models the matter dominated universe has an ever accelerating expansion contrary to the recent observations[6] as well as theoretical requirements[7]. In a recent work, however, it has been shown that along with a quintessence scalar field which

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interacts with the BD field, it is possible to have a scenario in which the quintessence field oscillates at an early epoch but grows later to drive the accelerated expansion for a fairly arbitrary set of quintessence potentials[8].

The second problem is of an entirely different nature. The dimensionless parameter ω in Brans-Dicke theory plays a crucial role in the prediction of observational results. Although the popular belief that BD theory goes over to GR in the infinite ω limit suffered a jolt[9], but in the weak field regime BD results get closer to GR results for higher values of ω . The local astronomical experiments are quite well explained by GR and demands a pretty high (a few hundreds) value of ω [10] if the predictions made in BD theory have to be within the observational uncertainty. On the other hand in most of the models in the Brans-Dicke framework, the accelerated expansion of the universe requires a very low value of ω , typically of the order of unity. However, a recent work shows that if the BD scalar field interacts with the dark matter, a generalised BD theory can perhaps serve the purpose of driving an acceleration even with a high value of ω [11].

In these investigations, either Brans-Dicke theory is modified to suit the present requirement or a quintessence scalar field is used to generate sufficient acceleration. In ref.[11] and in a recent work by Barrow and Clifton[12], no additional potential were added, but an interaction between the BD scalar field and the dark matter were used to do the needful.

It was also shown that a Brans-Dicke scalar field alone can drive an accelerated expansion in the matter dominated epoch, without any quintessence matter or any interaction between the BD field and the dark matter[13]. The problem once again was that it required a very low value of ω , of the order of unity, and there was no transition from a decelerated to an accelerated scenario.

In the present work, we intend to show that a generalisation of Brans-Dicke theory by Bergman and Wagoner[14] and in a more useful form by Nordtvedt[15] can in fact solve at least the first problem. In this generalisation, the parameter ω is taken to be a function of the BD scalar field instead of its being a constant. Different functional forms of ω could originate from various physical motivations. It is indeed an appealing feature of any model if the accelerated expansion can be generated without the requirement of an additional quintessence field. Naturally it would be interesting to check if some form of $\omega(\phi)$ can give rise to a decelerated expansion to start with and helps entering into an accelerated expansion phase later, but all in the matter dominated regime. In what follows we shall show that indeed a simple choice of ω as a function the Brans-Dicke scalar field does the trick.

In the next section the model with a variable ω is presented. Section 3 deals with some specific examples where the deceleration parameter has a smooth transition from a positive to a negative value and section 4 presents some discussions on the results obtained and the possibilities for some future work.

2 Field Equations

For a spatially flat Robertson Walker spacetime, the field equations in the generalised Brans-Dicke theory are,

$$3\frac{\dot{a}^2}{a^2} = \frac{\rho}{\phi} + \frac{\omega(\phi)}{2}\frac{\dot{\phi}^2}{\phi^2} - 3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}, \quad (1)$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{\omega(\phi)}{2}\frac{\dot{\phi}^2}{\phi^2} - 2\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi}, \quad (2)$$

where a is the scale factor of the universe, ρ is the density of the matter distribution, ϕ is the Brans-Dicke scalar field and ω the dimensionless parameter, now a function of ϕ rather than being a constant. The units are chosen where $8\pi G = 1$. The thermodynamic pressure of the cosmic fluid is taken to be zero consistent with the present dust universe. In what follows, we shall assume the conservation equation for matter leading to the relation

$$\rho = \frac{\rho_0}{a^3}, \quad (3)$$

where ρ_0 is a constant. Hence the wave equation for the scalar field ϕ ,

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = \frac{1}{2\omega + 3}(T - \dot{\phi}^2\frac{d\omega}{d\phi}), \quad (4)$$

is not an independent equation, and follows from the Bianchi identities. Here T is the trace of the energy momentum tensor for matter. So we have two equations (1) and (2) for three unknowns a , ϕ and ω . We choose the relation,

$$\phi = \phi_0 a^n, \quad (5)$$

so that the equation system is now closed. Here ϕ_0 and n are constants. There is however no apriori reason for this choice only except that the equation system becomes tractable and consistent. With this choice and equation (3), the system of equations can be readily integrated for a constant ω corresponding to Brans-Dicke theory. The scale factor $a(t)$ comes out as a simple power function of the cosmic time t and hence rules out the possibility of any transition of the deceleration parameter $q = -\frac{\ddot{a}a}{\dot{a}^2}$ from a positive to a negative value in the matter dominated epoch. The expansion can either be ever accelerating or ever decelerating depending on the choice of values of the constants. This is in perfect agreement with the work already in the literature[11] and [13]. In Nordtvedt's generalised theory with a varying ω , the situation can be dramatically different. Using relation (5) in a combination of equations (1) and (2), one can write

$$\frac{\ddot{a}}{a} + (n+2)\frac{\dot{a}^2}{a^2} = \frac{\rho_0}{\phi_0}a^{-(n+3)}, \quad (6)$$

which has a first integral of the form

$$\dot{a}^2 = \frac{2\rho_0}{\phi_0(n+2)(n+3)}a^{-(n+1)} + Aa^{-2(n+2)}, \quad (7)$$

A being a constant of integration. From equations (6) and (7), the deceleration parameter q can be written as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{A\phi_0(n+2)^2(n+3) + \rho_0(n+1)a^{(n+3)}}{A\phi_0(n+2)(n+3) + 2\rho_0a^{(n+3)}}. \quad (8)$$

Amongst the constants ρ_0 is positive definite and so is ϕ_0 as the Newtonian constant of gravitation and hence ϕ should always be positive. The constants A and n can have negative values as well and can help finding a q which gets into a negative value in a recent past, if certain conditions are satisfied.

3 A particular model

In what follows, we shall assume that A is positive but $-3 < n < -1$. This indeed gives a possibility that q starts positive for small a but eventually attains a negative value when

$$|n+1|\rho_0 a^{(n+3)} > A\phi_0(n+2)^2(n+3).$$

In order to get a flair for the numbers, we choose

$$n = -\frac{3}{2},$$

so that $q = \frac{1}{2}$ for a negligible value of a , i.e, we assume at the outset that the beginning of the matter dominated ($p = 0$) era behaves the same as that in spatially flat FRW model in general relativity. The relation between the constants will now determine the time at which q crosses the zero value. If we assume that $q = 0$ at $z = 1.5$ we get the relation

$$\frac{3A\phi_0}{4} = \rho_0(2/5)^{3/2}, \quad (9)$$

where the redshift z is given by

$$1+z = \frac{a_0}{a},$$

the subscript 0 indicating the present value. The deceleration parameter then has a simple form

$$q = \frac{1}{2} \left[\frac{1 - (\frac{5a}{2})^{3/2}}{1 + 2(\frac{5a}{2})^{3/2}} \right]. \quad (10)$$

We have scaled a such that its present value $a_0 = 1$. Thus, for $n = -\frac{3}{2}$, the deceleration parameter q is close to 0.5 for a very small value of a , becomes zero at $a = \frac{2}{5}$, i.e, $z = 1.5$ and has a negative value of $q \approx -0.16$ at the present epoch. Clearly, the time of transition of the signature of q is sensitive to the choice of the constants, and hence a fine tuning will enable us to get the correct epoch where q crosses the zero value.

Using equations (3), (5) and (7) in equation (1) with the choice $n = -\frac{3}{2}$, the functional dependence of the choice of $\omega(\phi)$ can be written as

$$\omega(\phi) = -\frac{4}{3} \left[1 + \frac{1}{2(2+\phi)} \right]. \quad (11)$$

Clearly ω has a negative value, which indicates where does the negative contribution to the effective pressure come from. For other choices of the constant n , the functional form of $\omega(\phi)$ will be different. Furthermore, the form of $\omega(\phi)$ also fine tunes the value of z at which q crosses the zero value in favour of a negative one. For instance, if $n = -\frac{3}{2}$ and $\omega(\phi)$ is given as

$$\omega(\phi) = -\frac{4}{3} \left[1 + \frac{\alpha}{2(\phi + 2\alpha)} \right], \quad (12)$$

where

$$\alpha = (2)^{3/2},$$

the signature flip in q takes place at $z = 1.0$. With the same form of $\omega(\phi)$ with $\alpha = (3/2)^{3/2}$, the flip takes place at $z = 0.5$.

In view of the high degree of non-linearity of Einstein's equations, it is now important to check whether the reconstruction of the form of $\omega = \omega(\phi)$ gives rise to the same form of q . In this work, the detailed stability analysis, presumably the subject matter for another full-fledged paper, is not carried out. But it can be said that the forms of $\omega(\phi)$ given are both necessary and sufficient for the corresponding behaviour of q . For example, a particular evolution for q given by equation (10) yields the form of $\omega(\phi)$ given by equation (11) showing the necessity of the latter to arrive at the form of q given by equation (10). However, if one now takes up (11) as the input and use equations (2) and (5), the same behaviour of q is obtained. This shows the sufficiency of equation (11). So long as the results of the stability analysis is not known, this necessary and sufficient nature of equation (11) shows that the solution does worth attention.

4 Discussions:

The present work clearly shows that a generalised scalar tensor theory where the BD parameter ω is a function of the scalar field ϕ , can drive an accelerated expansion for the present universe without having to resort to an additional quintessence field. Unlike most of the Brans-Dicke models, a varying ω even allows for a signature flip in q in the matter dominated epoch. This indeed requires a fine tuning of the parameters, but the merit of the model is that this transition can be shown analytically.

The value of ω , which effects this smooth transition, is not specified, only the functional form of ω can be determined. But this gives an advantage. For local astronomical experiments, ω can have a high value due to the local inhomogeneity as ϕ would be function of the space coordinates. But at a cosmological scale, the value of ω , averaged over the spatial volume of the universe, could be small and hence one can avoid the nagging problem of the discrepancy of the values of ω for a cosmological requirement and the local experiments.

The present work, however, has its own problems. Although the value of ω required is not specified, equation (12) indicates that it has a low negative value. This contradicts the local astronomical requirement of a high value of ω as mentioned earlier. Furthermore, a negative ω , particularly $\omega < -\frac{3}{2}$ leads to a negative contribution to the kinetic part of the energy leading to quantum instabilities[17]. However this problem is shared by most of the phantom models with a negative Hamiltonian. The form of $\omega(\phi)$ is chosen phenomenologically rather than inspired by any underlying physics. For that matter the quintessence potentials are all chosen like that so the present model is no worse than any of the quintessence models on the count of a sound theoretical basis. The advantage here is that the scalar field itself is already there in the purview of the theory and is not put in by hand.

The particular model presented in section 2 assumes ϕ as a power function of the scale factor as $\phi = \phi_0 a^n$, which yields

$$\frac{\dot{\phi}}{\phi} = nH.$$

As in this theory, ϕ is the inverse of the effective Newtonian constant G , thus one has

$$\frac{\dot{G}}{G} = -nH.$$

For this particular model to work efficiently one requires that n should be of the order of unity, so the fractional rate of variation of G is of the same order of magnitude as H . Observational limits indicate that the rate should be smaller[16]. Definitely one would have been more comfortable with values of n not greater than 10^{-1} , but this is only a primitive model, and the high degree of nonlinearity keeps the possibility of getting the required features of the model with other choices of $\omega(\phi)$ wide open. Surely investigations along this line, i.e, to find a form of $\omega(\phi)$ which preserves the features of this model and gives a better value of $\frac{\dot{G}}{G}$, is warranted.

The other problem of the model is quite generic for all the dark energy candidates, namely that of the fine tuning of parameter. One exception of this is of course the tracking solutions where the potential grows to drive acceleration in the later stages from a wide range of initial conditions [18].

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